

NON-COMMUTATIVE TOPOLOGICALLY MASSIVE GAUGE THEORY

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We investigate the perturbative dynamics of noncommutative topologically massive gauge theories with softly broken supersymmetry. The deformed dispersion relations induced by noncommutativity are derived and their implications on the quantum consistency of the theory are discussed.

1. Introduction

Non-commutative quantum field theory is a fascinating theoretical laboratory where highly non-trivial deformations of space-time structures induce novel and unexpected dynamical effects at quantum level. Recently they have attracted a lot of attention, mainly due to the discovery of their relation to string/M theory ^{1,2}. In particular, Seiberg and Witten ² realized that a certain class of quantum field theories on non-commutative Minkowski space-times can be obtained as a particular low energy limit of open strings in the presence of a constant NS-NS B -field. From a purely field theoretical point of view they appear as a peculiar non-local deformation of conventional quantum field theory, presenting a large variety of new phenomena not completely understood, even at perturbative level. Four dimensional non-commutative gauge theories are in fact afflicted by the infamous UV/IR mixing ³ that complicates the renormalization program and it may produce tachyonic instabilities ⁴. We will not try to address these problems in $D=4$ ⁵: our investigations will be instead concentrated on the three dimensional, non-commutative, topologically massive electrodynamics for a number of reasons. First of all the presence of a single physical polarization and of an explicit gauge-invariant mass for the photon should simplify the analysis of the UV/IR mixing and elucidate the nature of the

tachyonic instabilities. Secondly, planar non-commutative gauge theories with Chern-Simons terms have been proposed as effective description of the Fractional Hall Effect^{6,7}. Last but not least, two of us^a share with Stanley an insane passion for the ubiquitous Chern-Simons term and its unusual dynamical properties: we hope he will enjoy our non-commutative exercises on topologically massive gauge theory, of which he is a Master.

2. Non-commutative $U(1)$ Yang-Mills-Chern-Simons

2.1. *Pure gauge model, and its symmetries*

Non-commutative^b topologically massive $U(1)$ gauge theory in three dimensions is governed by the Lagrangian

$$S = -\frac{1}{4} \int d^3x F_{\mu\nu} \star F^{\mu\nu} - m_g \int d^3x \varepsilon^{\lambda\mu\nu} \left(\frac{1}{2} A_\lambda \star \partial_\mu A_\nu + \frac{ig}{3} A_\lambda \star A_\mu \star A_\nu \right), \quad (1)$$

where the field-strength is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]_\star \quad (2)$$

and the \star stands for the usual Moyal product,

$$(f \star g)(x) \doteq \int \frac{d^2y}{\pi\vartheta} \int \frac{d^2z}{\pi\vartheta} f(t, y) g(t, z) e^{-\frac{2i}{\vartheta} \varepsilon_{0ij} (y-x)^i (z-x)^j}. \quad (3)$$

Differently from the commutative case, which simply describes the propagation of a free massive boson, the new non-commutative incarnation is an interacting theory, resembling more a non-abelian model than an abelian one^c.

This richer structure at the level of interactions is however paid when considering the global symmetries. The constant tensor $\vartheta^{\mu\nu}$ present in the definition of the Moyal product does waste the original Lorentz invariance. [In three dimensions, there is no Lorentz-invariant constant antisymmetric two-tensor.] In the case of space-like non-commutativity ($\vartheta^{\mu\nu} \vartheta_{\mu\nu} < 0$), the residual symmetry can be identified with the spacial rotation $SO(2)$ and the translations. For time-like non-commutativity ($\vartheta^{\mu\nu} \vartheta_{\mu\nu} > 0$), $SO(2)$ is replaced by $SO(1,1)$, but the theory is not unitary⁸. Finally, dealing with

^aL.G. and D.S. plead guilty for leading astray the innocent souls of N.C. and S.P. with this project.

^bIn what follows we shall only consider the case of space-like and therefore the constant tensor $\vartheta^{\mu\nu}$ is chosen to be $\vartheta \varepsilon^{0\mu\nu}$, where the index 0 denotes the time component.

^cThis is not surprising, because the actual gauge group of (1) can be identified with a particular realization of $U(\infty)$.

light-like non-commutativity ($\vartheta^{\mu\nu}\vartheta_{\mu\nu} = 0$) is trickier, but one can show that the residual group is " \mathbb{R} ". The discrete symmetries (C, P, T) instead follow closely the known path of the commutative case: C is conserved, while P and T are again broken.

The equations of motion derived from the action (1), but not the action itself, are obviously gauge invariant against the \star -gauge transformation

$$A_\mu^u(x) \doteq u(x) \star A_\mu(x) \star u^\dagger(x) - \frac{i}{g} \partial_\mu u(x) \star u^\dagger(x), \quad (4)$$

generated by \star -unitary functions $u(x) \star u^\dagger(x) = u(x)^\dagger \star u(x) = 1$. The presence of the Chern-Simons term in (1) produces indeed a non-vanishing variation,

$$\begin{aligned} \delta S &= \frac{m_g}{6g^2} \int d^3x \varepsilon^{\lambda\mu\nu} (u^\dagger \star \partial_\lambda u \star u^\dagger \star \partial_\mu u \star u^\dagger \star \partial_\nu u) - \\ &\quad - \frac{m_g}{2g} \int d^3x \varepsilon^{\lambda\mu\nu} \partial_\lambda (u^\dagger \star \partial_\mu u \star A_\nu) = 4\pi^2 \left(\frac{m_g}{g^2} \right) w(u) + \text{total divergence} \end{aligned} \quad (5)$$

where $w(u)$ is the non-commutative version of the usual winding index. An example of transformation u , for which $w(u)$ is not zero, is given by

$$u_{\mathbb{P}} = [1 - \mathbb{P}(x, y)] + e^{ib(t)} \mathbb{P}(x, y), \quad (6)$$

where $\mathbb{P}(x, y) = 2 \exp(-(x^2 + y^2)/|\vartheta|)$ is a \star -projector ($\mathbb{P} \star \mathbb{P} = \mathbb{P}$) and $b(t)$ is any function such that $b(t)|_{-\infty}^{\infty} = 2\pi$. In this particular case, we find $w(u) = 1$. Thus, as occurs for non-abelian topologically massive gauge theory⁹, the consistency of the quantum theory requires that the mass m_g is quantized according to the relation

$$4\pi^2 \left(\frac{m_g}{g^2} \right) = 2\pi k. \quad (7)$$

2.2. The $\mathcal{N} = 1$ supersymmetric extension

At the perturbative level one of the most puzzling feature of non-commutative field theory is the phenomenon of the ultraviolet-infrared (UV/IR) mixing. The non-local nature of the interaction, while softening the behavior at large momenta, moves the UV divergences into the IR region. This effect generically endangers the stability of the perturbative vacuum, the unitarity and the infrared finiteness of the theory.

An elegant way to have under control these potential problems is to consider the supersymmetric extension of the model. Supersymmetry improving the ultraviolet behavior of a theory will also act, via (UV/IR) mixing, as an

infrared regulator. In fact, if the number of supersymmetries is sufficiently large, all the undesired divergences will disappear from the infrared region. In three dimensions, for the case of the Yang-Mills Chern-Simons (YMCS) system, it is enough to consider the $\mathcal{N} = 1$ extension of the model, whose Lagrangian is obtained by minimally coupling a Majorana fermion to the action (1),

$$S_{\text{S-YMCS}}^{\text{NC}} = -\frac{1}{4} \int d^3x F_{\mu\nu} \star F^{\mu\nu} + \frac{1}{2} \int d^3x \bar{\lambda} \star (i\not{D} - m_f) \star \lambda + \\ -\frac{1}{2} m_g \varepsilon^{\lambda\mu\nu} \int d^3x A_\lambda \star \partial_\mu A_\nu + \frac{i}{3} g m_g \int d^3x \varepsilon^{\lambda\mu\nu} A_\lambda \star A_\mu \star A_\nu. \quad (8)$$

In eq. (8) we have also softly broken the supersymmetry to $\mathcal{N} = 0$ by choosing different masses for the gauge field and the Majorana fermion. This will not jeopardize the cancellation of the infrared singularities because, in this case, they are related just to the leading ultraviolet divergences. Besides this breaking will provide us with a much richer and interesting model: by taking, in fact, different limits for the masses, we can focus our attention, for example, either on the pure bosonic theory ($m_f \rightarrow \infty$) or on the usual supersymmetric gauge theory ($m_f = 0$, $m_g = 0$) or on the Chern-Simons theory ($m_g \rightarrow \infty$).

Finally a remark is in order. Naively one may expect that there is no problem with the UV/IR mixing for the YMCS system. In fact topologically massive commutative gauge theories are super-renormalizable models, that actually result UV-finite in perturbation theory. Thus, apparently, there is no UV divergence to be moved in the IR region. However their finiteness originates partly from their symmetries: the simultaneous presence of Lorentz and gauge invariance forbids the potential linear divergences. In the non-commutative set-up Lorentz invariance is lost and the linear divergences will reappear as infrared divergences via (UV/IR) mixing. However the theory is still UV-finite.

3. The one-loop two-point function

The simplest way to address the question of vacuum stability and unitarity is to analyze the one-loop one-particle irreducible two-point function for the gauge boson. At the tree level, this function coincides with the commutative one since the \star -product is irrelevant in the quadratic part of the action

(1)^d. Its tree level form in the Landau gauge is in fact

$$\Gamma_{\mu\nu}^{\text{tree}}(p) = \eta_{\mu\nu}p^2 - p_\mu p_\nu - im_g \varepsilon_{\mu\nu\lambda} p_\lambda. \quad (9)$$

In the commutative case, when computing the one-loop correction, the only effect of the radiative corrections is to properly renormalize the two transverse structures in (9). In fact they can be recast in the general form

$$\Pi_{\mu\nu}(p) = \Pi_e(p)(\eta_{\mu\nu}p^2 - p_\mu p_\nu) - im_g \Pi_o(p) \varepsilon_{\mu\nu\lambda} p_\lambda. \quad (10)$$

The two functions Π_e and Π_o , computed in ^{9,10}, govern the commutative wave-function and the mass renormalization respectively.

This simple setting cannot be promoted to the non-commutative case as it originates from the simultaneous presence of gauge and Poincaré invariance which is now broken. Once the Lorentz invariance is lost, we cannot expect just one wave-function ($\mathcal{Z}_e = 1 - \Pi_e$) and mass ($\mathcal{Z}_m = 1 - \Pi_o$) renormalization, since different components of the gauge field may renormalize in different ways. More importantly, even the transversality of the one-loop correction to the $\Gamma_{\mu\nu}^{\text{tree}}$ may be endangered. This possibility, for example, takes place in the non abelian gauge theory at finite temperature ^{11,12}, where the space-time symmetries are destroyed by the choice of a preferred reference system, the thermal bath.

Therefore, before proceeding, we must carefully reexamine the Ward Identity that controls the longitudinal part of the $\Pi_{\mu\nu}$. A tedious exercise, with the non-commutative version of the BRST transformation, shows that in any covariant ξ gauge (and thus also in the Landau gauge) the following Ward identity holds

$$p_\lambda \Pi^{\lambda\alpha}(p) = g\Gamma_\nu(p) (p^\nu p^\alpha - p^2 \delta^{\nu\alpha} - im_g \varepsilon^{\nu\alpha\beta} p_\beta - i\Pi^{\nu\alpha}(p)) \quad (11)$$

where Γ_ν is defined through the following vacuum expectation $\langle \bar{c}(x)[A_\nu(y), c(y)]_\star \rangle_0 \equiv i \int d^3z \mathcal{G}(x - y - z) \Gamma_\nu(z)$, with \mathcal{G} the exact ghost propagator. In the commutative case Γ_ν is compelled by Lorentz invariance to be proportional to p_ν and the above identity entails transversality. In the non-commutative model, there are two new possible vectors that can appear in the expansion of Γ_ν ,

$$\tilde{p}^\mu = \vartheta^{\mu\sigma} p_\sigma \quad \chi^\mu = \varepsilon^{\mu\alpha\beta} \tilde{p}_\alpha p_\beta, \quad (12)$$

^dIt holds the following property:

$$\int f \star g = \int fg.$$

and the above argument seems to break. However a detailed one-loop analysis shows that Γ^ν has surprisingly no component along \tilde{p}^μ and χ^μ . Therefore the transversality is preserved at one-loop. At higher loops, the situation is less clear, but there are indications that this property is preserved. Once we have convinced ourselves that the transversality is kept, we can write the most general form for the $\Pi_{\mu\nu}$, which is also compatible with the bosonic symmetry

$$\Pi_{\mu\nu} = \Pi_1^e p^2 \frac{\chi_\mu \chi_\nu}{\chi^2} + \Pi_2^e \frac{\tilde{p}_\mu \tilde{p}_\nu}{\tilde{p}^2} p^2 - \Pi^o i m_g \varepsilon_{\mu\nu\lambda} p^\lambda + \bar{\Pi}^o (\tilde{p}_\mu \chi_\nu + \tilde{p}_\nu \chi_\mu). \quad (13)$$

Actually the last tensor structure will not appear at any order in perturbation theory because of the accidental invariance $\vartheta \rightarrow -\vartheta$ that $\Pi_{\mu\nu}$ possesses. This, combined with the Bose symmetry, implies that $\bar{\Pi}^o$ must be even in ϑ and odd in p but such a scalar cannot be built. We are left with

$$\Pi_{\mu\nu} = \Pi_1^e p^2 \frac{\chi_\mu \chi_\nu}{\chi^2} + \Pi_2^e \frac{\tilde{p}_\mu \tilde{p}_\nu}{\tilde{p}^2} p^2 - \Pi^o i m_g \varepsilon_{\mu\nu\lambda} p^\lambda. \quad (14)$$

At the end of the day the only effect of non-commutativity is to produce two different wave-function renormalization: one for the component $(\tilde{p} \cdot A)$ and one for the component $(\chi \cdot A)$. The commutative case (10) is recovered when $\Pi_1^e = \Pi_2^e$, because $\eta_{\mu\nu} - p_\mu p_\nu / p^2 = \chi_\mu \chi_\nu / \chi^2 + \tilde{p}_\mu \tilde{p}_\nu / \tilde{p}^2$. Summing the general form (14) of the radiative correction to the tree level contribution (9) and inverting the total result, we obtain the renormalized propagator

$$G_{\mu\nu}^R(p) = \frac{1}{\sqrt{\mathcal{Z}_1 \mathcal{Z}_2} [p^2 - (m_g^R)^2]} \left(\sqrt{\frac{\mathcal{Z}_2}{\mathcal{Z}_1}} \frac{\chi_\mu \chi_\nu}{\chi^2} + \sqrt{\frac{\mathcal{Z}_1}{\mathcal{Z}_2}} \frac{\tilde{p}_\mu \tilde{p}_\nu}{\tilde{p}^2} + i m_g^R \varepsilon_{\mu\nu\lambda} \frac{p^\lambda}{p^2} \right),$$

where

$$\mathcal{Z}_1 = 1 - \Pi_1^e \quad \mathcal{Z}_2 = 1 - \Pi_2^e \quad \mathcal{Z}_m = 1 - \Pi^o \quad (m_g^R)^2 = \frac{m_g^2 \mathcal{Z}_m^2}{\mathcal{Z}_1 \mathcal{Z}_2}.$$

In next section, by looking at different features of $G_{\mu\nu}^R(p)$ at one-loop, we shall illustrate how the non-commutativity affects the spectrum of the theory, its unitarity and its vacuum stability. But for accomplishing that, we need the explicit form of scalar functions Π_1^e , Π_2^e and Π^o , whose evaluation is lengthy and tedious. In the following we shall not report on the details of the computations, which will appear in ¹³. The final result is given for completeness in appendix A. Here we shall limit ourselves to some general comments on their properties. Each function displays two contributions, which originates respectively from the "planar" and "non-planar" diagrams. The former is identical to the commutative (non abelian) case, while the

latter carries the effects of the non-commutativity. They are both finite and they cancel each other when ϑ goes to zero. This decoupling occurs since the softly broken supersymmetric model smoothes the effect of the UV/IR mixing. Finally both contributions possess a physical threshold at $p^2 = 4m_g^2$, but two unphysical threshold at $p^2 = 0$ and $p^2 = m_g^2$. The last feature will complicate our future analysis.

4. Dispersion relation and the stability of the vacuum

The spectrum of the non-commutative Yang-Mills Chern-Simons system is entirely encoded in the poles of the above propagator^e. Firstly it contains an unphysical pole at $p^2 = 0$, which describes the longitudinal degree of freedom still propagating in any covariant gauge. Secondly, it contains the relevant physical pole at

$$p^2 = (m_g^R)^2(p, \tilde{p}) = \frac{m_g^2 \mathcal{Z}_m^2(p, \tilde{p})}{\mathcal{Z}_1(p, \tilde{p}) \mathcal{Z}_2(p, \tilde{p})}, \quad (15)$$

which represents the effect of the radiative corrections on the tree level pole at $p^2 = m_g^2$. Since the Lorentz invariance is broken, eq. (15) does not depend only on p^2 but also on the new invariant \tilde{p}^2 , which is simply proportional to the euclidean norm of the spacial momentum for the case of space-like non-commutativity. Therefore the pole condition (15) should not be thought as an equation for evaluating the radiative corrected mass, but rather as an equation that determines the energy of the excitation in terms of its momentum, namely the dispersion relation. In a relativistic theory, this question is pointless because the functional form of the dispersion relation is fixed by the Poincaré symmetry.

The simplest way to solve eq. (15) is to proceed perturbatively. At the lowest order in $\left(\frac{g^2}{m_g}\right)$ we have:

$$E^2 = \tilde{p}^2 + m_g^2 \left[1 - \left(\frac{g^2}{m_g}\right) \left(2\Pi^o(p, \tilde{p}) - \Pi_1^e(p, \tilde{p}) - \Pi_2^e(p, \tilde{p}) \right) \right]_{p^2=m_g^2}, \quad (16)$$

where we have factored out the dependence of the one-loop Π s on the coupling constant. In order that eq. (16) provide a reasonable dispersion relation for a stable physical excitation, two criteria must be met: (a) it has to be gauge invariant; (b) it has to be real. These two requirements

^eSimilar investigations has been performed in ⁷

are far from being manifest, since the explicit form of the Π s is plagued by many complex contributions (see appendix A) coming from the unphysical thresholds at $p^2 = 0$ and $p^2 = m_g^2$ and moreover our perturbative computation has been performed in the Landau gauge.

The first point can be easily clarified by evaluating the combination $2\Pi^o(p, \tilde{p}) - \Pi_1^e(p, \tilde{p}) - \Pi_2^e(p, \tilde{p})$ at the threshold $p^2 = m_g^2$. A series of unexpected cancellations occur and the final result is completely real. This apparent miracle is just a signal that unitarity is preserved. The above combination can be in fact reinterpreted as the S-matrix element describing the transition from one particle state to one particle state¹⁰. Thus, if unitarity is not violated, this element must be free from unphysical cuts.

The interpretation of the combination $2\Pi^o(p, \tilde{p}) - \Pi_1^e(p, \tilde{p}) - \Pi_2^e(p, \tilde{p})$ for $p^2 = m_g^2$ as an element of the S-matrix also solves the second puzzle. In fact we know that S-matrix elements are gauge invariant. An alternative proof can be also given by means of the Nielsen identity.

For space-like non-commutativity, the explicit form of the gauge boson dispersion relation (16) reads

$$\begin{aligned} \frac{E_g^2}{m_g^2} = & 1 + \frac{\mathbf{p}^2}{m_g^2} + \frac{g^2}{8\pi m_g} \left\{ (1 + 2\mu)^2 \left(\int_0^1 dx \frac{e^{-\xi\sqrt{\mu^2 - x + x^2}}}{\sqrt{\mu^2 - x + x^2}} - \log\left(\frac{2|\mu| + 1}{2|\mu| - 1}\right) + \frac{1}{|\mu|} \right) \right. \\ & \left. - 27 \left(\int_0^1 dx \frac{e^{-\xi\sqrt{1 - x + x^2}}}{\sqrt{1 - x + x^2}} - \log 3 \right) + 4 \left(\frac{e^{-|\mu|\xi} - e^{-\xi}}{\xi} - \frac{1 + 4\mu + 4|\mu|}{4|\mu|} \right) \right\}, \quad (17) \end{aligned}$$

where we have introduced the dimensionless variables $\xi = m_g \tilde{p}$ and $\mu = m_f/m_g$ for convenience. In this dispersion relation we can distinguish essentially three terms: the first bracket contains the fermion contribution, the second parenthesis collects instead the gauge contribution, while the last piece is the remnant of the UV/IR mixing. This expression is in fact finite in the infrared region $\xi \rightarrow 0$ for finite μ . In the mere supersymmetric case $\mu = 1$ eq. (17) dramatically simplifies and we are left just with the bosonic contribution, but with a different coefficient: -18 instead of -27 . If $\mu \rightarrow \infty$, i.e. if we approach the pure Yang-Mills Chern-Simons system, the UV/IR mixing will rise again. In fact the last bracket will produce an infrared divergent term of the form $-4e^{-\xi}/\xi$. The rising of this negative divergent contribution at small ξ for sufficiently large μ will always make the square of the energy negative in a certain region of the spacial momenta (see fig. 1). In other words, the massive excitation becomes a tachyon and the perturbative vacuum is no longer stable. Varying the other two parameters g^2/m_g and $m_g^2\vartheta$ will not affect the picture: it will only change the

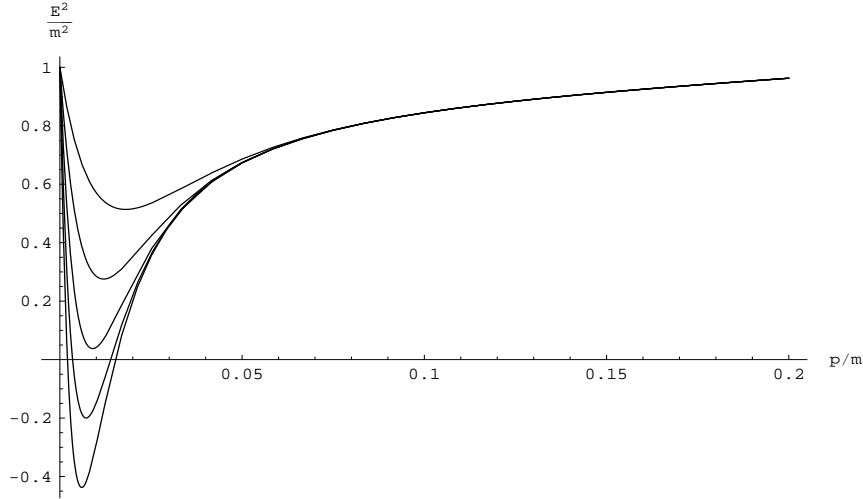


Figure 1. The dispersion relation of the boson for small spacial momenta, when $g^2/m_g = 0.1$ $m_g^2 \vartheta = 1$ and $\mu = 100 \rightarrow 300$.

specific value of μ at which the tachyon will appear. Thus, when we reach the critical value of μ , we must resort with non-perturbative techniques to select the new vacuum. At the moment, the nature of this new vacuum is only matter of speculation⁷. One may conjecture that the transition tuned by the tachyonic mode will lead the system to a sort of stripe phase analogous to that proposed by Gubser and Sondhi¹⁴ for φ^4 . But this possibility is quite problematic: a non translationally invariant vacuum would mean a dynamical breaking of the gauge invariance and this could endanger the consistency of the entire theory. Recall that, for a non-commutative gauge theory, space-time translations are in fact a subset of the gauge transformations.

A less speculative point of view, but nevertheless very intriguing, is to suppose that the tachyonic mode will drive the Yang-Mills Chern-Simons system through a phase transition similar to the one speculated by Cornwall¹⁵ for the non abelian model, in the commutative case, in the large N -limit. We must recall in fact that there is a great similarity between non-commutative gauge theories and gauge theories at large N ¹⁶.

The fate of the perturbative vacuum should be discussed, of course, at non-perturbative level: a possibility is by employing the matrix model representation of the theory and it will be the object of future investigations.

Appendix A. Analytic expression of the different Π s

If we introduce the following basic integrals ($k = -1, 0, 1$)

$$T_k^{(\text{np})}(\mu_1, \mu_2) = \frac{\partial^{k+1}}{\partial \xi^{k+1}} \left(\int_0^1 dx \frac{(-1)^{k+1} e^{-\xi \sqrt{x(x-1)\eta^2 + x\mu_1^2 + (1-x)\mu_2^2}}}{\sqrt{x(x-1)\eta^2 + x\mu_1^2 + (1-x)\mu_2^2}} \right) \quad (\text{A.1})$$

and the dimensionless variable $\eta^2 = p^2/m_g^2$, the non-planar contributions for the different Π s are given by

Gluon Sector:

$$\begin{aligned} \Pi_{1,\text{np}}^e = & -\frac{g^2}{8\pi m_g} \left[\left(\frac{9(4-\eta^2)}{4} T_{-1}^{(\text{np})}(1,1) - \frac{(4-5\eta^2+\eta^4)}{\eta^2 \xi} T_0^{(\text{np})}(1,1) \right) \right. \\ & + \left(\frac{5(1-\eta^2)^2}{2\eta^2} T_{-1}^{(\text{np})}(1,0) - \frac{(6-2\eta^2)}{\xi} T_0^{(\text{np})}(1,0) \right) - \left(\frac{\eta^2}{4} T_{-1}^{(\text{np})}(0,0) - \right. \\ & \left. \left. - \frac{(1-\eta^2)}{\xi} T_0^{(\text{np})}(0,0) \right) + \frac{(9-4\eta^2)e^{-\xi} - (5-4\eta^2)}{\eta^2 \xi} \right] \quad (\text{A.2}) \end{aligned}$$

$$\begin{aligned} \Pi_{2,\text{np}}^e = & -\frac{g^2}{8\pi m_g} \left[\left(\frac{9(4-\eta^2)}{4} T_{-1}^{(\text{np})}(1,1) + \frac{(4-5\eta^2+\eta^4)}{\eta^2 \xi} T_1^{(\text{np})}(1,1) \right) \right. \\ & + \left(\frac{5(1-\eta^2)^2}{2\eta^2} T_{-1}^{(\text{np})}(1,0) + \frac{(6-2\eta^2)}{\xi} T_1^{(\text{np})}(1,1) \right) - \left(\frac{\eta^2}{4} T_{-1}^{(\text{np})}(0,0) + \right. \\ & \left. + \frac{(1-\eta^2)}{\xi} T_1^{(\text{np})}(0,0) \right) + \frac{(5-4\eta^2) - (9-4\eta^2)e^{-\xi}}{\eta^2 \xi} \right] \quad (\text{A.3}) \end{aligned}$$

$$\begin{aligned} \Pi_{\text{np}}^o = & -\frac{g^2}{16\pi m_g} \left[\left(\frac{3}{2}\eta^4 - 2\eta^2 \right) T_{-1}^{(\text{np})}(0,0) - \left(3\eta^2 - \frac{3}{2}\eta^4 + 12 \right) T_{-1}^{(\text{np})}(1,1) \right. \\ & \left. - \frac{(1-\eta^2)^2(1+3\eta^2)}{\eta^2 \xi} T_{-1}^{(\text{np})}(1,0) + 2 \frac{(1-2\eta^2)}{\eta^2} \frac{1-e^{-\xi}}{\xi} \right] \quad (\text{A.4}) \end{aligned}$$

Fermion Sector:

$$\begin{aligned} \Pi_{1,\text{np}}^e = & -\frac{g^2}{2\pi m_g \eta^2} \left(T_1^{(\text{np})}(\mu, \mu) - \mu^2 T_{-1}^{(\text{np})}(\mu, \mu) \right), \quad \Pi_{\text{np}}^0 = -\frac{g^2 \mu}{4\pi m_g} T_{-1}^{(\text{np})}(\mu, \mu), \\ \Pi_{2,\text{np}}^e = & -\frac{g^2}{2\pi m_g \eta^2} \left(\mu^2 T_{-1}^{(\text{np})}(\mu, \mu) + \frac{1}{\xi} T_0^{(\text{np})}(\mu, \mu) \right). \quad (\text{A.5}) \end{aligned}$$

The planar contribution are identical to that of Pisarski and Rao ¹⁰.

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